Precautionary Protectionism

Sharon Traiberman*       Martin Rotemberg†

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Abstract

Should tariffs be used to protect industries that are crucial in times of national crises? In this paper we show that in a Ricardian setting the answer is no. We develop a dynamic extension of Dornbusch et al. (1977) with adjustment costs: the home country is less productive in the short run for goods it is not initially producing. If the home country would not produce a good in the absence of adjustment costs, then industrial policy should not encourage its production. Instead, we find that the optimal policy is to protect goods where comparative (dis)advantage is only weak, particularly for goods where protection induces production or exporting. The intuition is that this type of protection strategy effectively frees up Foreign labor for the production of crucial goods.

1 Introduction

It is unclear if the Coronavirus crisis reflects a higher risks of pandemics going forward. It is clear that those risks are now more salient, prompting discussions of appropriate industrial policy. In the popular press, a common argument is that countries should protect industries which are likely to be important in a crisis (such as masks or vaccine inputs). To understand the economic forces at play, we build on the Ricardian model of Dornbusch et al. (1977) to study precautionary trade policy in a world with crises and adjustment costs. We consider adjustment costs with the following form: not producing a good today means that productivity for the good will be lower tomorrow. We focus on “foreign-biased” demand shocks: sharp increases in demand for goods which the home country has a comparative disadvantage.

While we do find a role for protectionism, it does not coincide with imposing tariffs for the specific products that are needed in times of crisis. Comparative advantage is largely the guiding force for trade and trade policy, reflecting results from the static case (Opp, 2010; Costinot et al., 2015). The logic is straightforward: if there are products which are particularly important in a crisis, it is valuable for policymakers in the home country to encourage their production in the place where it can be done at the lowest cost, which may be abroad. The role for protectionism is to improve the terms of trade, lowering the domestic price of the newly valuable goods. To do this, the optimal policy is to encourage the domestic production of products where the home country is only marginally less productive than the foreign country, which effectively frees up foreign labor.

*New York University. st1012@nyu.edu
†New York University. mr3019@nyu.edu
2For instance, see https://americanaffairsjournal.org/2020/05/reshoring-supply-chains-a-practical-policy-agenda/.
Formally, our approach adapts the set-up in Costinot et al. (2015) to consider production dynamics. We consider adjustment costs on the extensive margin: if in steady-state it takes $a_i$ workers to produce a unit of good $i$, it takes $a_i(1 + x)$ workers in the short run for goods that are not initially produced. We think this assumption reflects the state of the policy debate: should the United States should promote mask and ventilator production at home, so that it can be scaled up quickly in a crisis, since the cost of scaling up discontinuously increases at the extensive margin.

While perhaps the optimal solution would be to produce an $\epsilon$ amount of everything (just in case), this isn’t a policy lever available to the government: the only tool available is tariffs. If a tariff is high enough to encourage domestic production, firms will endogenously choose to satisfy all of domestic demand.\footnote{While we do not model this explicitly, we implicitly assume that free entry and productivity spillovers mean that firms themselves do not internalize the dynamic productivity benefits of production (Hausmann and Rodrik, 2003; Mussa, 1982).} It is useful to benchmark our results to the case with no adjustment costs. This scenario implies no need for forward-looking policy making, and, as in Costinot et al. (2015), comparative advantage is all that matters. Domestic-production follows a cut-off rule in (relative) productivity, with the home country producing every good whose ranking below the cut-off and none above. A foreign-biased demand shock raises the cutoff in the second period (to a product where the home country is relatively less productive).

Adding adjustment costs, we start by describing the deterministic two-period case, when the policy maker knows what sectors will be affected in the following period. We then turn to richer models, and find that our main conclusions are similar. If a product wouldn’t be produced domestically in a crisis using steady-state technologies, there is no precautionary reason for policy to reshore production (similarly, all goods who would originally be produced at home still should be). In all cases, as in Costinot et al. (2015), we find that the optimal tariff on imported goods in each period is uniform.

We find more nuance between the pre and post crisis cutoffs: the goods which would not be produced at the initial steady-state optimum, but would be produced in a crisis steady-state. In particular, the existence of a strict cut-off in productivity depends on both the nature of the adjustment costs and on the dynamics of demand.

In our benchmark case, we consider extremes: infinite adjustment costs (so there can be no short-run production of new goods), and a secular shift in demand of all relevant goods in the range (so the crisis corresponds to a large increase in demand for goods that are always being produced abroad, and a proportional decline in demand for the rest). Here, there will be a single cut-off in production, which corresponds to an average of the initial and final steady-state cutoffs.

In this benchmark case, we can split the region between those cutoffs into two sub-regions. The boundary between the sub-regions corresponds to the good where foreign consumers are indifferent between local or imported goods (because the ratio of the wages is equal to the ratio of unit labor requirements). In one sub-region (the one corresponding to lower comparative advantage), in the second period unprotected goods will be imported, but protected goods will only be consumed domestically: they are only cheaper at home because of the tariff. This sub-region corresponds to the endogenous non-traded region in Dornbusch et al.
The other sub-region (to the left of the boundary), is new to our dynamic framework. In the second period, unprotected goods will be imported, and protected goods will be exported.

As $x$ falls, the number of sub-regions increases (up to four). In order of increasing comparative advantage, the first two sub-regions are as described above. The next least relatively productive sub-region corresponds to goods where the home country will produce no matter what, but will only be productive enough to export if the good is protected in the first period. Finally, the most productive sub-region comprises goods which the home country will export no matter which technology is used. Within each sub-region there is a cutoff that is monotonic in comparative advantage, but goods may be protected in a sub-region even when goods with higher comparative advantage (in a different sub-region) are unprotected, and therefore not produced in the first period.

Allowing for flexible demand shocks also induces non-monotonic cutoffs: it is particularly valuable to protect a good whose demand will increase substantially in the crisis. The logic is straightforward: it is particularly valuable to onshore production of sectors where (a) the difference in country productivity is small and (b) the demand is particularly high in the pandemic state.

We build on the Dornbusch et al. (1977)-with-tariffs frameworks of Opp (2010) and Costinot et al. (2015) to add in dynamics on the demand side. This complements a large literature studying dynamics on the production side, such as Beshkar and Shourideh (2020), Naito (2019), and Matsuyama (1992). Our main contribution to this literature is considering adjustment costs, although we do not specifically micro-found the adjustment costs as in Leamer (1980), and Furusawa and Lai (1999).

In Section 2, we describe the environment, and define the no-tariff equilibrium for each period. In Section 3, we introduce demand shocks and frictions. In Section 4 we solve for optimal policy.

2 Model

2.1 Environment

We consider the Ricardian environment of Dornbusch et al. (1977), augmented for trade policy as in Costinot et al. (2015). There are two countries, Home and Foreign, a continuum of goods indexed by $i \in [0, 1]$, and two periods, $t = 1, 2$. Foreign variables are denoted by an asterisk in all subsequent discussion. Home and Foreign are each endowed with an unchanged amount of inelastically supplied labor, $L$ and $L^*$, respectively.

2.2 Preferences

There is a representative worker-consumer in each country. Statically, preferences are given by a Cobb-Douglas aggregator over goods:

$$u_t = \exp \left\{ \int_0^1 \beta_t \log(c_{it})di \right\}, \quad (1)$$
where $c_i$ is consumption of good $i$. The $\beta$ terms are expenditure shares for each good and are susceptible to shocks over time. For simplicity, we assume that $\beta_{it} = \beta_{it}^*$. The consumer has log preferences over intertemporal utility and discounts at the rate $\delta$. Hence, the utility for the household is given by,

$$U = \log(u_1) + \delta \log(u_2) \quad (2)$$

### 2.3 Technology and Adjustment Costs

The only input into production is labor. Unit input requirements for each good are given by $a_{it}$. We will order goods so that $A(i) \equiv a_{i1}^* / a_{i1}$ is decreasing in $i$. This ratio is the **comparative advantage schedule**. We assume throughout that $A$ is smooth and strictly decreasing.

Let $m_{i1}$ be a dummy for importing in period 1. Technology evolves according to the following assumption:

**Assumption 2.1. External Economies of Scale:**

$$a_{i2} = \begin{cases} 
a_{i1} & \text{if } m_{i1} < 1 \\
a_{i1}(1 + x) & \text{if } m_{i1} = 1.
\end{cases} \quad (3)$$

$$a_{i2}^* = a_{i1}^* \quad (4)$$

This equation states that at home, if production does not occur in period 1, then producing the good in period 2 involves an additional amount of labor $x$. Our benchmark case will is where $x = \infty$, so if Home does not produce in the first period, it cannot produce in the second. With intuition from the benchmark in hand, we discuss the case where $x$ is finite. There are two simplifications embedded in this setup, both of which we discuss later in the paper. First, we assume that adjustment costs are only incurred at home not in Foreign. The situation that interests us is one in which Home’s terms of trade deteriorate in response to a shift in demand to Foreign. Reducing Foreign’s comparative advantage, without allowing them to retaliate, counteracts this shift in a way that complicates expressions and places restrictions on the parameter space without changing any major results. Second, we assume that $x$ is constant across goods. As we will see in the expressions that follow, this is not necessary, but keeps the exposition from becoming cumbersome.

### 2.4 Market Structure and Trade

Markets are perfectly competitive and workers are freely mobile between sectors. Let $w$ and $w^*$ be the wage paid to workers in Home and Foreign respectively. Then the unit cost of producing $i$ is $wa_i$ and $wa_i^*$, respectively. An important assumption is that while the planner has the preferences of the consumers, firms in our model optimize statically:

**Assumption 2.2. Myopia:** Firms set $\delta = 0$.

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\(^4\)This can be thought of as a combination of the probability of the pandemic and the discount factor.
This assumption allows us to ignore how much firms internalize external economies of scale. This is an important assumption, as M. Grossman and Rossi-Hansberg (2010) have shown that in the DFS model, the extent to which firms internalize externalities is determined by industry structure, and need not be zero.

The only trade barriers we consider are (weakly) positive tariffs $t_{it}$ enacted by the Home government. Given the assumption on trade, consumers source from the lowest cost producer. Hence, $m_{it}$, the import status of good $i$ at home in time $t$, is given by,

$$m_{it} = \arg\min \{ w_{at}, w^*_{at}(1 + t_{it}) \} \quad (5)$$

### 2.5 Equilibrium

An equilibrium in this model is a set of wages, $\{w_t, w^*_t\}$, prices, $\{p_{it}, p^*_{it}\}$, labor allocations, $\{l_{it}, l^*_{it}\}$, consumption decisions, $\{c_{it}, c^*_{it}\}$, tariffs, $\{t_{it}\}$, and import decisions, $\{m_{it}, m^*_{it}\}$ such that:

1. Households in home (and similarly in foreign) maximize (1) subject to the budget constraint:

$$\int_i p_{it} c_{it} \leq w_L + \int_i p^*_{it} c_{it} m_{it} t_{it} di.$$

2. Prices are set competitively, $p_{it} = \min \{ w_{at}, w^*_{at}(1 + t_{it}) \}$

3. Markets clear:

$$c^*_{it}(1 - m_{it}) + c^*_{it} m^*_{it} = l_{it}/a_{it}$$

4. Allocations are feasible:

$$\int_i a_{it} l_{it} di \leq L$$

5. Trade is balanced:

$$\int_i p_{it} c^*_{it} m^*_{it} di = \int_i p^*_{it} c_{it} m_{it} di$$

Solving this equilibrium given tariffs is straightforward and follows Dornbusch et al. (1977), adjusted for the presence of tariffs. In particular, once $A$ is sorted in each period, Foreign will use a cutoff to decide what to import given by solving $A(i) = \omega^5$. In this case, the equilibrium conditions become:

$$\omega_t = \ell \frac{\int_0^A \beta^{A^{-1}(\omega^*)} \beta_{it} di} {\int_0^A \beta_{it} m_{it} t_{it} di} \times \left( 1 - \frac{\int_i \beta_{it} m_{it} t_{it} di} {1 + t_{it}} \right) \quad (6)$$

$$m_{it} = \arg\min \{ \omega_t, A^*_t(1 + t_{it}) \} \quad (7)$$

where $\omega$ is the relative wage of home to foreign, $w/w^*$, and $\ell$ is the ratio of population on foreign to home, $L/L^*$. WLOG, we set $w^*$ to numeraire, so that $\omega$ is the wage at home. The left hand side of equation (6) is

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\[\text{We have written this so that A is invertible in a neighborhood of the equilibrium in both cases. It is easy to check if Foreign’s cutoff in period 2 is the same of Home’s in period 1—where there is now a jump discontinuity.}\]
clearly strictly increasing in $\omega$. Moreover, one can show that the right hand side, paired with (7) is decreasing in $\omega$. In the special case that tariffs are uniform, so that Home uses a simple cutoff for importing, we can write the above as a function of Foreign’s cutoff, $t^F \equiv A(\omega)$, and the real wage:

$$\omega = \frac{\ell \int_0^{t^F} \beta t dt}{\int_1^{\tau} \beta t dt} \times \left( 1 + \tau \int_0^{t^H} \beta t dt \right),$$

(8)

where $t^H$ is given implicitly by $A^{-1}(A(t^F)(1 + \tau))$. The left hand side is now a decreasing function of $t^F$ and the right hand side is an increasing function of $t^F$. Where the function $A$ and $\Phi$ intersect is the equilibrium outcome. Indeed, for the case that $\tau = 0$ this is exactly Dornbusch et al. (1977).

Figure 1: A Foreign Biased Demand Shock

3 Foreign Biased Demand Shocks

To focus on the situation where Home wants to expand production after a shock, we assume that the distribution of $\beta$ shifts for high $i$ goods. In particular, we assume that for some $i$ that is above the period 2 free trade equilibrium, $\int_1^{t_2} \beta_{i2} > \int_1^{t_1} \beta_{i1}$. We call this a foreign biased demand shock. To understand what happens with such a shock, consider the case of free trade with $t_{it} = 0$. Then, in each period imports take a
simple cutoff rule and equation (6) reduces to the standard system of equations from DFS:

\[
A_t(\bar{\iota}) = \bar{\omega} \\
\frac{\ell \int_0^1 \beta_{it} dt}{\int_1^1 \beta_{it} dt} = \bar{\omega}.
\]

If the distribution of \(\beta\) shifts to the right, then for a fixed \(\iota\), \(\omega\) would decrease. Thus, the initial \(\iota\) could not be an equilibrium, and would have to rise to offset the drop in \(\omega\). Figure 1 plots the free trade equilibrium, holding \(A\) fixed, for two different schedules of \(\beta\). The variables denoted with a prime are after the foreign biased shock.

As can be seen in the figure, a foreign biased shock with free trade will increase the number of products that Home makes domestically, and lower Home’s wage. Intuitively, Foreign labor will be allocated to the high \(i\) goods, and home picks up the slack. The home worker operating in the marginal industry is now less productive, while the opposite is true of the foreign workers, which leads to a lower relative wage at home.

What happens with a foreign biased demand shock when there are short run adjustment frictions? This situation is plotted in figure 2. The dotted lines now represent the first period, while the solid lines, with primed functions, the second. In the case of free trade, the second period comparative schedule is zero above the cutoff, and equal to its initial value before. Meanwhile, \(\Phi\) shifts to the right, as in figure 1. However, because technology is fixed, the resulting equilibrium in period 2 is different. In particular, \(\bar{\iota}_2 = \bar{\iota}\) instead of \(\bar{\iota}'\), as before. This in turn makes Home even worse off, as their relative wage declines even further on account of the adjustment frictions.

**Figure 2: Foreign Biased Demand Shock with Frictions**
Home’s wage declines on account of misallocation—Home is stuck making low demand goods. While Foreign is relatively more well off than Home, one can show they are also worse off than in the equilibrium without adjustment frictions, for a similar reason. The size of the misallocation effect depends on the shock.

Home therefore has two possible motivations for protection. The first, unrelated to the above discussion, is the static terms of trade motivation as discussed in Opp (2010) and Costinot et al. (2015) (CDVW). However, there is now a dynamic tradeoff: misallocation must occur on account of the adjustment frictions, so how much should the planner shift this to the first period in order to improve the terms of trade in the second? We turn to this question now.

4 Optimal Protectionism

4.1 Problem Setup

In this section we define the Home government’s problem, and characterize optimal policy. As in CDVW, we focus on the primal problem of the planner. First we solve for allocations while taking account of consumers’ decision rules, and then we show how to decentralize this. We also focus on characterizing optimal tariffs as a function of the real wages, focusing on the case that $\omega_2 < \omega_1$. At the optimum, this will hold as Home is strictly worse off in the second period for any configuration of $m_{i1}$ and $\beta_i$, given the assumptions on technology and demand shocks. As the planner cannot bootstrap a better situation, there is no sense in considering the case where she may try.

The planner’s problem is to maximize home utility subject to optimality by Foreign consumers and Home consumers, as well as subject to feasibility constraints. Given the assumptions on Foreign, they are relatively passive and follow a cutoff rule in each period for determining their import behavior. However, their behavior in the second period may be endogenous to the choices of Home, even holding fixed wages. With Cobb-Douglas preferences, expenditure shares are given by $\beta_{it}$. Detailed derivations are left to the appendix.

We begin with a discussion of the planner’s full problem, which is given by:

$$V(w_t, w_t^*) = \max_{c_{it}, m_{it}, \tau_{it}} L \int \beta_{i1} \log(c_{i1}/L)di + \delta L \int \beta_{i2} \log(c_{i2}/L)di$$

subject to

$$\int_0^1 a_{it} c_{it}^*(1 - m_{it}^*)di + \int_0^1 a_{it} c_{it} m_{it}di \leq L^*$$

$$\int_0^1 a_{it} c_{it}^* m_{it}^*di + \int_0^1 a_{it} c_{it}(1 - m_{it})di \leq L$$

$$c_{it} = \beta_{it}(w_tL + T_i)/\min\{w_ta_{it}, w_t^*a_{it}^*(1 + \tau_{it})\}$$

$$m_{it} = \arg\min\{w_ta_{it}, w_t^*a_{it}^*(1 + \tau_{it})\} ,$$

$$c_{it}^* = \beta_{it} w_t^* L^*/\min\{w_t^*a_{it}^*, w_t a_{it}\}$$

$$m_{it}^* = \arg\min\{w_t^*a_{it}^*, w_t a_{it}\} ,$$
where \( a_{i2} \) is a function of \( m_{i1} \). To economize on space, we have omitted the market clearing condition and definition of \( T_t \). These constraints are redundant in the case of Cobb-Douglas preferences, as they are implied by the feasibility constraints and optimal sourcing and consumption rules.

In order to solve this problem, we follow CDVW and momentarily ignore the constraints on \( c_{it} \) and \( m_{it} \). We call this the relaxed problem, and it can be written as the following Lagrangian:

\[
L(\omega_1, \omega_2) = \int_0^1 \left[ \beta_{i1} \log c_{i1}/L + \delta \beta_{i2} \log c_{i2}/L + \nu_1(L^* - a_{i1}^* c_{i1}^*(1 - m_{i1}^*) - a_{i1}^* c_{i1} m_{i1}^*) + \nu_1(1 - a_{i1} c_{i1}(1 - m_{i1})) + \delta \nu_2(L^* - a_{i2}^* c_{i2}^*(1 - m_{i2}^*) - a_{i2}^* c_{i2} m_{i2}) + \delta \nu_2(1 - a_{i2} c_{i2}(1 - m_{i2})) \right],
\]

where \( m_{i2}^* \) are the (known) import decision rules for Foreign. The optimization strategy relies on the fact that any solution to \( (RP) \) that is feasible in \( (PP) \), must also solve \( (PP) \).

### 4.2 Benchmark: Infinite Adjustment Frictions

In our benchmark case, we will assume that \( x = \infty \), that \( \beta_{i1} = 1 \) and that \( \beta_{i2} = \lambda < 1 \) for \( i < i^* \) and is equal to \( \frac{1 - \lambda i}{1 - \lambda} \) otherwise. We will also assume that \( \nu^* \) is large in a way we will make precise shortly. The first assumption implies that if the planner does not produce the good in the first period, the good must be imported in the second. Similarly, Foreign will produce the good domestically. The second assumption places enough structure on the schedule of \( \beta \) that one can use the Leibniz rule to differentiate integrals of the form \( \int_{f(z)}^g(\beta) di \) w/r/t \( z \). Nevertheless, in order to later discuss the case of unrestricted \( \beta \)’s, and to make clear when this smoothness is necessary, we will continue to write \( \beta_{it} \) as a parameter.

Turning to the optimization, we can maximize this Lagrangian by doing so for each \( i \). Notice that if \( m_{it} \) is known, then this is concave in \( c_{it} \) and can be maximized the standard way. In particular, at any solution for import rules,

\[
\frac{\partial L}{\partial c_{it}} : c_{it} = \frac{\beta_{it} L}{\nu^* a_{it}^* m_{it} + \nu a_{it}(1 - m_{it})}.
\]

This equation tells us that for the solution to the relaxed problem to be an optimum, it will require that \( \tau_{it} \) is uniform on imported goods. One can see this by taking the ratio for any two import goods implied by this equation,

\[
\frac{c_{it}}{c_{i't}} = \frac{\beta_{it}}{\beta_{i't}} \times \frac{a_{i't}^*}{a_{it}^*},
\]

which matches with the consumer’s decision rule iff \( 1 + \tau_{it} = 1 + \tau_{i't} \). Now we will show that this uniformity is still implied by the optimal decision rules for imported goods, but that in the first period the planner will set prohibitive tariffs when the uniform tariff is too low. Hence, the set of goods will not be determined by a cutoff rule that depends only on \( A \), as in CDVW or Opp (2010).

To analyze import behavior, we work backwards. In the second period, the value function is differentiable
in $m_{i2}$, treated as a variable on $[0, 1]$. The associated first order condition is,

$$
\frac{\partial L}{\partial m_{i2}} : m_{i2} = 1 \Leftrightarrow -\nu_2^* a_i^* + \nu_2 m_{i2} > 0.
$$

This equation states that the planner will import or produce at home depending on whether $\frac{a_i^*}{a_{i2}} < \frac{\nu_2}{\nu_2^*}$. This solution to (RP) will be a solution for (PP) only if $\tau_2 = \frac{\nu_2}{\nu_2^*} - 1$. This also implies a uniform tariff in period 2. Since $a_{i2} = \infty$ if $m_{i1} = 1$, the optimal importing strategy in period 2 will be given by:

$$
m_{i2} = \begin{cases} 
1 & \text{if } m_{i1} = 1 \\
0 & \text{if } m_{i1}=0 \text{ and } A_i \leq \nu_2/\nu_2^* 
\end{cases}
$$

(10)

Foreign’s import strategy will similarly be given by,

$$
m_{i2}^* = \begin{cases} 
0 & \text{if } m_{i1} = 1 \\
1 & \text{if } m_{i1}=0 \text{ and } A_i > \omega_2 
\end{cases}
$$

(11)

Thus, given $\nu_2/\nu_2^*$, we have shown that the import strategy in period 2 is merely a function of period 1’s import strategy. Let $\nu_1 = A^{-1}(\nu_2/\nu_2^*)$ be the cutoff for Home, and $\nu_2 = A^{-1}(\omega_2)$ be the cutoff for Foreign.

Turning to the first period, the Lagrangian is no longer differentiable. Hence, while for any choice of $m_{i1}$, we can solve for consumption and period 2 policies, we cannot use first order methods to deduce the optimal policy in period 1. However, the linearity of $L$ in $i$ greatly facilitates analysis. In particular, we can optimize good-by-good to solve this problem, taking account of the optimal choice of $c_{i1}$, $m_{i2}$, and $m_{i2}^*$ conditional on $m_{i1}$.

Let $L_i^{m_{i1}, m_{i2}}$ the contribution to the Lagrangian of good $i$ given choices of $m_{i1}$, $m_{i2}$, and $m_{i2}^*$. One can show that given the optimal consumption decision,

$$
L_i^{m_{i1}, m_{i2}, m_{i2}^*} = \kappa_i - \beta_i \log(a_{i1} \nu_1 (1 - m_{i1}) + a_i^* \nu_i^* m_{i1}) - \delta \beta_i \log(a_{i2} \nu_2 (1 - m_{i2}) + a_i^* \nu_i^* m_{i2})
$$

$$
- \delta \beta_i^2 \nu_2^* (1 - m_{i2}^*) + \nu_2 w_2^*/\nu_2 m_{i2}^*,
$$

(12)

where $\kappa_i$ is a constant independent of any choices. Since $m_{i2}$ and $m_{i2}^*$ have been solved for conditional on $m_{i1}$, this is really just a function of $m_{i1}$. This formulation of the problem in the first period naturally lends itself to a solution strategy: every good will fall into a partition of the unit interval given by $[0, \nu_1]$, $[\nu_1, \nu_2^*]$, $[\nu_2^*, \nu_2]$, and $[\nu_2, 1]$, which dictate the second period decisions of a given first period decision; hence, in each region the planner makes a binary decision over $m_{i1}$.

We depict this situation in figure 3. The ordered triples in each region are the options the planner considers. In the first and fourth regions, the second period actions are independent of the first period actions, and so it’s clear what the planner will do—export those goods to the left of $\nu_1$ and import those
to the right of $i^H_2$. Intuitively, there is no “disagreement” for these goods between the two periods. Notice that if $i^*$ is larger than the optimal cutoff in the absence of adjustment frictions, then the planner will never import these goods. And so we assume $i^*$ is at least this large. We leave the derivations to the appendix, but under our benchmark case we prove the following:

**Proposition 4.1.** For a given choice of wages, the optimal policy is given by:

\[
m_{i1} = 1 \iff A_1 \leq \begin{cases} \frac{v_1}{v_1^*} & \text{if } i \in [0, i^H_1] \\ \left(\frac{v_1}{v_1^*}\right)^{\frac{1}{1+\delta}} \times \left(\frac{v_2}{v_2^*}\right)^{\frac{1+\delta}{1+\delta}} \times \exp \left( -\frac{\delta}{1+\delta} \frac{L}{L^*} \Phi_{H,t} \left( 1 - \frac{v_2/v_2^*}{\omega_2} \right) \right) & \text{if } i \in [i^H_1, i^F_2] \\ \left(\frac{v_1}{v_1^*}\right)^{\frac{1}{1+\delta}} \times \left(\frac{v_2}{v_2^*}\right)^{\frac{1+\delta}{1+\delta}} & \text{if } i \in [i^F_2, i^H_2] \\ \left(\frac{v_1}{v_1^*}\right)^{\frac{1}{1+\delta}} & \text{if } i \in [i^H_2, 1] \end{cases}
\]  

(13)

where $\Phi_{H,t}$ and $\Phi_{F,t}$ are Home and Foreign import expenditure shares. Moreover, there exists a map $T(v_1/v_1^*)$ that is monotonic in the ratio of multipliers and has a fixed point at the optimum.

Here we present a sketch of the proof. First note that the cutoff rules follow from manipulating (18). Each piece has an economic interpretation. The ratios of the multipliers are the statically optimal cutoffs for each period—they denote the bounds on the comparative advantage schedule where Home is either too productive or too unproductive for dynamic considerations to matter. This echoes CDVW’s finding of an optimal cutoff. The middle regions first contain the geometric mean of the two cutoffs weighted by the demand shock—reflecting that the planner combines the two cutoffs, using the demand shock to determine

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\[6\] Formally, one must check that since $A_1 > v_1/v_1^* > v_2/v_2^*$ in the first region and the opposite in the fourth, the planner does indeed choose to produce in the former and import in the latter.
importance. The final term, in region 2, reflects the term that comes from the ability to induce Foreign to export. Notice that in the interior region, the cutoff in the second region is smaller than in the third, but \( A \) is decreasing—hence, only one cutoff will ever be binding.

One issue with equation (13) is that it is defined implicitly by the import shares in period 2, which are a function of the import shares in period 1. Nevertheless, we prove that this problem has a fixed point for any choice of the ratio of multipliers. In fact, one can reach the fixed point by iterating on the import shares. Once we prove that the policy function exists, we know that it is also continuous and differentiable in the ratios of the multipliers. This can be seen by writing the import shares as functions of the cutoffs. Assuming the relevant cutoff is in region 2, the import share for Home in period 2 is given by,

\[
\Phi_{H,2} = 1 - \lambda A^{-1} \left( \frac{\nu_1}{\nu_1^*} \right)^{-\frac{\delta}{1+\delta}} \times \left( \frac{\nu_2}{\nu_2^*} \right)^{-\frac{\delta}{1+\delta}} \times \exp \left( - \frac{\delta \lambda L}{1 + \delta \lambda L} \Phi_{H,2} \left( 1 - \frac{\nu_2/\nu_2^*}{\omega_2} \right) \right).
\]

In this region, there are no non-traded goods, and so \( \Phi_{F,2} = 1 - \Phi_{H,2} \). Hence, one can use the implicit function theorem to solve for the derivative of the ratio of import shares. For the case that the cutoff is in region 1, it is obvious that imports are increasing in the ratio of the multipliers because there is no longer the implicit piece.

To complete the proof, we first show, given the derivative above, that import shares Home in period 2 are increasing in \( \nu_2/\nu_2^* \), and vice versa for Foreign. Moreover, the ratio of the multipliers is also monotonic in these import shares.\(^7\) Hence, for any given \( \nu_1/\nu_1^* \), the mapping that takes \( \nu_2/\nu_2^* \) to an import policy and then to a new value of the multipliers using the import policy rule, is monotonic in the ratios. This is a strict generalization over CDVW since the technology is endogenous. With this in hand, it’s clear that the optimal \( \nu_2/\nu_2^* \) is monotonically increasing in \( \nu_1/\nu_1^* \), since Home will weakly import more with worse technology. Then the argument can be repeated in the first period.

At this point, we can stress the content of our assumption on \( \beta_{it} \). The crux of proving existence comes down to needing imports to be increasing in the values, \( \nu_2/\nu_2^* \). At first glance, it may seem that this must be true: raising the multipliers is akin to lowering a tariff, which ought to increase imports. But recall that the policy function itself is implicit in the imports, and a higher import share decreases the incentive to import goods in period 1. Why? Because if the planner is importing more, it places pressure on Foreign workers to produce less demanded goods. The planner can maintain its desired terms of trade while lowering misallocation by inducing Foreign to purchase some of the goods at which Home is relatively productive in period 2 if they protect in period 1. With \( \beta \) free, one can still show that \( \Phi_{H,t} \) exists, but there is no guarantee that it is differentiable, and we would need to assume that the cutoff rules are increasing in \( \nu_2/\nu_2^* \).

\(^7\)One can show from the constraints that,

\[
\frac{\nu_t}{\nu_t^*} = \omega_t \times \frac{1 - \Phi_{H,t}}{\Phi_{H,t}} \times \frac{\Phi_{F,t}}{\nu_t^* L_t} - \Phi_{F,t}.
\]

This function is decreasing in Home’s import share, and increasing in Foreign’s import share. Hence, when \( \nu_2/\nu_2^* \) increases in policy function, the new value of \( \nu_2/\nu_2^* \) will decrease.
The situation where this might be violated is if, at a given $\nu_2/\nu_2^*$, the marginal good in region two is a very high $\beta_{1,2}$ good and so raising the cutoff just a bit, and starting to import this good, actually results in implicitly lowering the cutoff. In this situation, there is no guarantee of monotonicity of import shares in the multipliers, and thus no guarantee of existence. Hence, our approach may not work. That said, the economic content of our predictions do not change with variable $\beta$’s—the planner will still only consider protection for the intermediate region, but the policy may feature many jumps. With the intuition for how we approach the problem in hand, we present an extension below to the case where $x < \infty$, and then discuss the implications of our findings for trade policy in the face of adverse demand shocks.

4.3 Finite Adjustment Frictions

Adding finite adjustment costs is no different than having infinite adjustment costs, except that the potential number of regions that the planner must consider in the first period changes. To see this, first note that consumption is exactly the same as before. Moreover, the second period is almost exactly the same as before except that when $x = \infty$, neither Home nor Foreign will purchase the good in the second period if it is imported in the first. However, with finite $x$, there may be goods that Home or Foreign will purchase, even if Home pays the adjustment frictions. This will occur if $A_i/(1 + x) \geq \nu_2/\nu_2^*$ for Home, or larger than $\omega_2$ for Foreign. This defines two new cutoffs in period 2: $\iota_2^{H,x}$ solves $A(i) = (1 + x)\frac{\nu_2}{\nu_2^*}$ and is the cutoff for goods operating with adjustment costs (i.e., $m_{i1} = 1$); $\iota_2^{F,x}$ solves $A(i) = (1 + x)\omega_2$. We can still use equation (18) to solve this problem, except now $a_{i2} = a_i(1 + x)$ if $m_{i1} = 1$.

Figure 4 plots the new set of regions the planner must consider. The new red curve is the comparative
advantage schedule facing the adjustment frictions. The values of \( \nu_1/\nu_1^* \), \( \omega_2 \), and \( \nu_2/\nu_2^* \), partition the space into the intervals discussed above—the second period terms determine the second period decisions as an outcome of first period decisions; the first period term is the static cutoff in the first period. The new regions may actually be empty if \( \iota H_1 \) is large enough to shift the new cutoffs to its left, and in this case the adjustment frictions case collapses back to the previous case. We leave the technical details to the appendix, but summarize the expanded proposition here:

**Proposition 4.2.** For a given choice of wages, if the optimal policy exists it is given by:

\[
m_{i1} = 1 \Leftrightarrow A_i \leq \begin{cases} 
\frac{\nu_1}{\nu_1^*} & \text{if } i \in [0, \iota_1^H] \\
\frac{\nu_1}{\nu_1^*} \times (1 + x)^{-\delta \lambda} \times \exp \left( -\frac{\delta \lambda}{1 + \delta \lambda} \frac{L}{L^*} \Phi_H \left( 1 - \frac{\nu_2}{\nu_2^*} \right) \right) & \text{if } i \in [\iota_1^H, \iota_2^F, \iota_2^H] \\
\frac{\nu_1}{\nu_1^*} \times \left( \frac{\nu_1}{\nu_1^*} \right)^{\frac{1}{1 + \delta \lambda}} \times \exp \left( -\frac{\delta \lambda}{1 + \delta \lambda} \frac{L}{L^*} \Phi_H \left( 1 - \frac{\nu_2}{\nu_2^*} \right) \right) & \text{if } i \in [\iota_2^F, \iota_2^H] \\
\frac{\nu_2}{\nu_2^*} & \text{if } i \in [\iota_2^H, 1]. 
\end{cases}
\]

Moreover, for a given choice of the ratio of multipliers, import shares exist, and are continuous and differentiable. Finally, there are bounds on \( x \) for which the solution is guaranteed to exist.

The proof of this proposition proceeds exactly as in the proof of the original. Here there is an extra term, in the third region, that also pushes against monotonicity of import shares in the ratio of the multipliers. One can however verify by taking the derivative of \( \Phi_H \) that it will be monotonically increasing.

Having shown that the optimal policy is a mixture of uniform tariffs on goods that are imported, and prohibitive tariffs on a subset of goods in the first period, we have reduced the problem to simply solving for \( \nu_1/\nu_1^* \), which is a monotonic function of itself (through the import shares). Hence, similar to the finding in CDVW, despite starting out as an infinite dimensional problem, the actual optimal policy of the planner can be reduced to a single parameter optimization. However, our model has several new implications for thinking about protectionism as industrial policy in the face of a Foreign biased demand shock. We conclude this section with several remarks summarizing our findings.

**Remark 4.3.** Demand only matters when comparative disadvantage is weak.

This is the most important result that we find. The range for which demand shocks bite is an intermediate range, and is bounded above by the *laissez-faire* import cutoff. Specifically, the planner will always use comparative advantage at the extremes to determine whether to produce at home or abroad. Given that in the unconstrained case, they would still find it unprofitable to deviate from importing from foreign, this will remain the case. Only in an intermediate range—where there is essentially disagreement between period 1 and period 2 on what the unconstrained optimal policy would be—does the planner let demand determine how to balance the two periods.
Remark 4.4. For high comparative advantage goods, adjustment costs directly enter decision making. Otherwise, the only role of adjustment costs is to determine the regions where there is a tradeoff.

This is obvious from (14), but worth highlighting. In fact, it is most instructive in the case that $x \to \infty$, which shuts down the region $[\iota_1^H, \iota_2^{H,x}]$. Nevertheless, there are still times that the planner will face a dynamic trade off. However, since they will never produce with the inferior technology, the trade off is in finding the least resource cost way of maintaining a particular relative wage. For $x < \infty$, $x$ matters only for high comparative advantage goods, where Home will produce in the second period regardless of first period action.

Remark 4.5. The cutoff rules need not be monotonic in $i$, and thus not monotonic in comparative advantage.

While in the case of infinite adjustment costs, there is only one binding cutoff, this is not true with finite adjustment costs. If one considers the second and third region, holding demand fixed, the cutoff is actually tighter in the third region—hence importing is less likely, despite comparative advantage being weaker. Indeed, depending on the shape of $A_i$, it could be that one ultimately exports the good just to the left of $\iota_2^{F,i}$, but imports the good just to the right. The intuition here is that in one case, Foreign will buy no matter what, but in the other case, there is extra motive to produce at Home, as you can induce Foreign to import from you. This inducement creates a new motive for protection in the first period, akin to an export subsidy for the second period.

What are the lessons for optimal trade and industrial policy in a pandemic? Surprisingly, despite the fact that we have shown demand shocks can matter a great deal, it is unlikely that in a pandemic that greatly favors Foreign, there is much Home can do. If the shock dramatically reduces demand for goods at which Home is only marginally unproductive (small $\delta \lambda$), they cannot exploit this to improve their standing, as Foreign does not want to buy what they are selling. At the other extreme that $\delta \lambda$ is large, so that the pandemic is not too big of a shock to demand, then the cutoffs will be so close that the range of goods the planner worries about is small. It is only in an intermediate range, where the pandemic hurts demand for Home’s goods, but not so much that Home loses almost all power, that the planner may meaningfully exploit trade policy in order to keep domestic production alive. Moreover, if Home has a comparative advantage in those goods that are in demand, our analysis is irrelevant, and laissez faire equilibrium actually favors Home. Pushing against this however, is the notion that if pandemics become more likely ($\delta \to 1$), then there is substantial room for the planner to engage in proactive industrial policy. However, we stress that in the case that $\delta$ is large, it is also unlikely that our assumption on firm myopia remains a reasonable one.

4.4 Stochastic Demand Shocks

Finally, we turn to the case of stochastic demand shocks (Eaton and Grossman, 1985). We will return to the simplified case that $x$ is infinite, but it should be clear from the preceding discussion that finite adjustment
frictions are not substantively different, but introduce more cutoffs. So, suppose that while the planner
knows \( \delta \) and \( x \), there is uncertainty of \( \lambda \). Formally, let \( s \in S \) index the set of possible states and let \( \lambda_s \)
be the expenditure coefficients in each state, and let \( G(s) \) be the cumulative distribution function on \( S \).
We will assume that all relevant integrals exist and that every realization of \( s \) is a foreign biased demand
shock.\(^8\) In order to economize on space, we will omit details of derivation. Nevertheless, it is clear that static
consumption decisions are unchanged, and that the period 2 policy is unchanged as a function of \( m_{i1} \), but is
now also indexed by \( s \). As discussed above, for \( i \) such that \( A(i) \geq \nu_1/\nu_1^* \), the planner will always produce at
home. Furthermore, let \( \bar{T} \leq 1 \) solve \( \sup_s \{ A(i) = \nu_2(s)/\nu_2^*(s) \} \). Then for \( i > \bar{T} \) the planner will clearly always
import.

For \( i \in [i_1^H, \bar{T}] \) there is the possibility of the planner choosing to produce domestically or at home, depending
on demand conditions. Now there will be three regions to consider. The first region will be the case that
\( i \in [i_1, \ell_F^L(s)] \), the second will be that \( i \in [\ell_F^L(s), \ell_F^H(s)] \), and finally there will be the region \( i \in [\ell_F^H(s), \bar{T}] \).
This last region is new, and is the region where if the planner produces in the first period, in the second period
they will still import—this was ruled out before, but could happen in a probabilistic setting. I.e., the
planner could make a mistake ex-post. Denote the regions by \( R_1(s), R_{II}(s) \), and \( R_{III}(s) \) respectively, and
let \( S_j(i) = \{ s \in S : i \in R_j(s) \} \), be the set of realizations for which good \( i \) is in set \( j \). These are mutually
exclusive regions so let \( p_{ji} \), be the probability of being in that region, so that \( \sum p_{ji} = 1 \), and let \( E_{ji}(\cdot) \) be
the conditional expectation operator over \( S_j(i) \).

We can transform equation (18) into an expectation in the first period and rearrange to arrive at the
following cutoff rule for goods:

\[
m_{i1} = 1 \iff \frac{\log \left( \frac{\nu_1}{\nu_1^*} \right) + p_{I,i} \delta E_{I,i}(\lambda \log \left( \frac{\nu_2}{\nu_2^*} \right) + \lambda \frac{\partial \nu_2}{\partial \nu_2 \log E} \left( 1 - \frac{\nu_2}{\nu_2^*} \right) + p_{II,i} \delta E_{II,i}(\lambda \log \left( \frac{\nu_2}{\nu_2^*} \right))}{p_{I,i} \delta E_{I,i}(\lambda) + p_{II,i} \delta E_{II,i}(\lambda)} \]

(15)

While this equation seems cumbersome, it is simply the weighted average of the various cutoffs from
(13). The presence of the expectation prevents exponentiation from simplifying the expression as in the
non-stochastic case. However, the right hand side of the inequality is the probability weighted average of
the expected cutoff for region \( I \), and the expected cutoff for region \( II \). One interesting point is that these
weights do not sum to 1, albeit there is no such down-weighting on the first period cutoff term. This reflects
the fact that with probability \( p_{III} = 1 - p_I - p_{II} \), if the planner produces in period 1, they will make a
mistake ex post, as they will import anyway. Wanting to avoid this tilts the planner towards placing more
weight on the first period. That said, without more information on \( G \), it is difficult to know if uncertainty
makes the planner more or less likely to use industrial policy than if the shock is \( E(\lambda) \) with probability 1.
This because there are ultimately two forces at play: there is the standard precautionary motive that comes
from the planner’s desire to avoid risk stemming from the log term, and there is the desire to avoid mistakes.

\(^8\)This is actually irrelevant, but obviously some probability that the demand shock favors Home will militate against any
precautionary action.
5 Conclusion

Our analysis is based on the Dornbusch et al. (1977) set-up, and includes some simplifying assumptions that are important to highlight. First, the technological constraints we have assumed are that any production is enough to maintain the original productivity level. It is likely that intensive margin adjustments are also costly, due to short-run external diseconomies of scale, and the policy makers might be able to induce small amounts of production. Similarly, we have also ignored supply chains, focusing on trade in final goods. That said, we suspect that even in these situations the intuition on our set-up will still hold: trade policy should encourage production of goods in the location of their comparative advantage. A additional caveat is that we have ignored any kind of strategic interaction or bargaining between countries. This might be the most important extension, since provisions for emergencies could be an important part of international agreements, and ex-post countries have discussed export-bans. This is, for instance, an important motivation for the protection of war-related goods.

In this paper, we have considered the problem of a planner in a two country, two period world, facing a demand shock in the second. Due to adjustment costs and myopia on the part of producers, there is value for the planner to use trade policy to promote production in the first period, in order to facilitate better terms of trade in the second. We think of this as a reasonable approximation to how a planner may think of a pandemic or other large crisis: there is a small probability of large demand shocks occurring, and (potential) producers are unlikely to internalize this global risk ex ante. We show that the existence of adjustment costs doesn’t change the upper and lower bounds of production: in both periods the planner produces goods for which it has a strong comparative advantage, and imports goods in which its comparative disadvantage is large. In between this range, we find more nuance: the planner bases the decision (of whether to produce at home, using prohibitive tariffs, or to buy from abroad) on a weighted average of the rules that the planner would statically use in period 1 or period 2. The implications for this characterization carry several lessons for policy makers considering designing policy for crises, in particular the value of identifying the products that are on the margin of being produced domestically or exported. We provide a graphical representation of exactly where those products lie in comparative advantage space.

We show that the intuition is similar if the specific products that will be needed (and the extent of the change in their demand) are unknown. For most goods, comparative advantage plays the only role. There is still an intermediate range, and here the goods that are likely to be protected are those goods in the intermediate zone that are in high demand, relative to steady state, in the same states of the world as particularly large Foreign-biased demand shocks, or which are in relative high demand in all possible states of the world. This result solidifies the intuition that the role of home production in the intermediate region is predominantly about improving the terms of trade with the least misallocation of workers.
References


A Technical Details

A.1 Setting Up The Relaxed Problem

The full problem of the planner is given by,

\[
V = \max_{w_t, w_t^*, c_{it}, m_{it}, \tau_{it}} \int_i u_1(c_{i1}) di + \delta \int_i u_2(c_{i2}) di
\]

subj. to

\[
\int_0^1 a_i c_{i1}^* (1 - m_{i1}^*) di + \int_0^1 a_i^* c_{i1} m_{i1} di \leq L^*
\]

\[
\int_0^1 a_i^* c_{i2}^* (1 - m_{i2}^*) di + \int_0^1 a_i^* c_{i2} m_{i2} di \leq L^*
\]

\[
\int_0^1 c_{i1}^* w_1 a_i^* (1 - m_{i1}^*) di + \int_0^1 c_{i1}^* w_1 a_i m_{i1} di \leq w_1^* L^*
\]

\[
\int_0^1 c_{i2}^* w_1 a_i^* (1 - m_{i2}^*) di + \int_0^1 c_{i2}^* w_1 a_i m_{i2} di \leq w_2^* L^*
\]

\[
\int_0^1 a_i c_{i1}^* m_{i1}^* di + \int_0^1 a_i c_{i1} (1 - m_{i1}) di \leq L
\]

\[
\int_0^1 a_i c_{i2}^* m_{i2}^* di + \int_0^1 a_i c_{i2} (1 - m_{i2}) di \leq L
\]

\[
c_{it} = \beta_{it} \frac{w_t L + T_t}{\min \{w_t a_{it}, w_t^* a_{it}^*(1 + \tau_{it})\}},
\]

\[
m_{it} = \arg \min \{w_t a_{it}, w_t^* a_{it}^*(1 + \tau_{it})\},
\]

\[
c_{it}^* = \beta_{it} w_t^* L / \min \{w_t a_{it}, w_t^* a_{it}^*\},
\]

\[
m_{it}^* = \arg \min \{w_t^* a_{it}, w_t a_{it}\},
\]

and the home budget constraint will hold by Walras’s law. First we get to the planner’s problem of the text.

To do so, we first consider breaking up the optimization problem into an outer and an inner problem.

The inner problem will fix wages, and solve for allocations and tariffs conditional on this choice. Then we can optimize over wages given the solution to the problem. Two things to notice before proceeding. First, if foreign optimality holds then the foreign budget constraint holds automatically. This is because the Cobb-Douglas assumption ensures that whatever the optimal sourcing strategy of Foreign, they will spend a constant expenditure share $\beta_{it}$ on good $i$ at $t$. Hence, the budget constraint can be dropped as long as we keep the optimality restrictions on Foreign’s consumption and sourcing rules. Second, given wages in the first period, Foreign’s decision rule in the first period is pegged down independently of Home’s actions. In particular, they will use a cutoff rule to determine their import behavior, solving $A(i_F^1) = \frac{w_{i1}^*}{w_{i1}}$, where $A(i) = a_{i}^*/a_{i}$. Hence, we can drop this constraint as well. In the second period, Foreign’s sourcing decision is a function of Home’s sourcing decisions because the comparative advantage schedule becomes endogenous.
With these fixes in mind, we can write the inner problem as:

\[
V(w_t, w^*_t) = \max_{c_{it}, m_{it}, \tau_{it}} \int_0^1 u_{i1}(c_{i1}) di + \delta \int_0^1 u_{i2}(c_{i2}) di \\
\text{subj. to } \int_0^1 a^*_t c_{i1}^*(1 - m_{i1}) di + \int_0^1 a^*_t c_{i1} m_{i1} di \leq L^* \\
\int_0^1 a^*_t c_{i2}^*(1 - m_{i2}) di + \int_0^1 a^*_t c_{i2} m_{i2} di \leq L^* \\
\int_0^1 a_t c_{i1}^* m_{i1}^* di + \int_0^1 a_t c_{i1} (1 - m_{i1}) di \leq L \\
\int_0^1 a_t c_{i2}^* m_{i2}^* + \int_0^1 a_t c_{i2} (1 - m_{i2}) di \leq L \\
c_{it} = \beta_{it}[w_t L + T_i]/\min \{w_t a_{it}, w^*_t a^*_t (1 + \tau_{it})\} \\
m_{it} = \arg \min \{w_t a_{it}, w^*_t a^*_t (1 + \tau_{it})\},
\]

where we have not written out the constraints for \(c_{i2}^*\) and \(m_{i2}^*\), but understand them to be functions of \(m_{i1}\).

Our goal is to solve the inner problem and then maximize over wages, subject to market clearing.

To make progress on the inner problem we follow CDVW and focus on the relaxed planner’s problem, dropping the constraints on \(c_{it}\) and \(m_{it}\). We will then construct a solution to this relaxed problem and show that it is feasible in the full inner problem. The relaxed problem Lagrangian is thus given by:

\[
L(w_t, w^*_t) = \int_0^1 \left[ \beta_{i1} \log c_{i1} + \delta \beta_{i2} \log c_{i2} + \nu_1^*(L^* - a_{i1}^* c_{i1}^*(1 - m_{i1}^*)) - a_{i1}^* c_{i1} m_{i1} + \nu_1(L - a_{i1} c_{i1} m_{i1} - a_{i1} c_{i1} (1 - m_{i1})) + \delta \nu_2^*(L^* - a_{i2}^* c_{i2}^*(1 - m_{i2}^*)) - a_{i2}^* c_{i2} m_{i2} + \delta \nu_2(L - a_{i2} c_{i2} m_{i2} - a_{i2} c_{i2} (1 - m_{i2})) \right],
\]

where \(\delta^{-1} \nu_t\) and \(\delta^{-1} \nu^*_t\) are the multipliers on the feasibility constraints. We refer to each term in the integrand as the good-specific Lagrangian and denote it \(L_i\).

### A.2 Proof of Propositions 1 and 2

Since Proposition 1 is a special case 2, we solve the more general case here. To solve this problem, first focus on consumption. The objective is concave in \(c\) conditional on \(m\), and the first order condition implies,

\[
\frac{\beta_{it}}{c_{it}} = \nu_t a_{it} (1 - m_{it}) + \nu^*_t a^*_t m_{it}.
\]

Before turning to the import decision, we will also retrieve the value for the foreign multiplier. If we plug in that for Home’s imports, \(c_{it} = \beta_{it}/(a^*_t \nu^*_t)\) and that for Foreign’s domestic production, \(c^*_t = \beta_{it} w^* L^*/(a^*_t w^*_t) = \)
\[ \beta_{it} L^* / a^*_it, \text{ then from the feasibility constraint on Foreign we have,} \]
\[
\int_0^1 a^*_{it} \frac{\beta_{it} L^*}{a^*_{it}} (1 - m^*_{it}) + \int_0^1 a^*_{it} \frac{\beta_{it}}{a^*_{it} \nu^*_t} m_{it} du = L^*
\]
Rearranging, and using the fact that \( \int \beta_{it} di = 1 \), yields,
\[ \nu^*_t = \frac{1}{L^*} \times \frac{\int_0^1 \beta_{it} m_{it} di}{\int_0^1 \beta_{it} m^*_{it} di}. \] (16)
I.e., the Foreign multiplier is the ratio of Home’s import share to Foreign’s import share. By a similar calculation,
\[ \nu_t = \frac{1 - \int_0^1 \beta_{it} m_{it} di}{L - L^* / \omega_t \times \int_0^1 \beta_{it} m^*_{it} di}. \] (17)
From the first order condition on consumption, notice that for any two imported goods,
\[ \frac{c_{it}}{c_{i't}} = \frac{a^*_{it}}{a^*_{i't}}. \]
From consumer optimality, this is equal to,
\[ \frac{c_{it}}{c_{i't}} = \frac{a^*_{it}(1 + \tau_{it})}{a^*_{i't}(1 + \tau_{i't})}. \]
Hence, if the solution to the relaxed problem solves the inner problem, it must do so with a uniform tariff on imported goods.

Turning to the import decision, we work backwards first. We could proceed as in CDVW and take the FOC with \( m_{i2} \), treating it as if it is a continuous choice. We could also evaluate the Lagrangian conditional on the choice of consumption and \( m_{i1} \). In either case, the optimal decision rule is given by the inequality:
\[ m_{i2} = 1 \iff \frac{a^*_{i1}}{a^*_{i2}} \leq \frac{\nu_t}{\nu^*_t}. \]
Given \( m_{i1} \), this corresponds to two simple cutoff rules. For \( i \) such that \( m_{i1} = 1 \), the cutoff solves \( A(i^H_{2t})/(1 + x) = \frac{\nu_t}{\nu^*_t} \) (goods below the cutoff are produced at home, above are imported). For \( i \) such that \( m_{i1} = 0 \), the cutoff solves \( A(i^H_{2t}) = \frac{\nu_t}{\nu^*_t} \).
This cutoff rule will correspond to consumer optimality if we set the uniform tariff to solve,
\[ \frac{\nu_2}{\nu^*_2} = \frac{w_2}{w^*_2(1 + \tau_2)}. \]
I.e.,
\[ \tau_2 = \frac{w_2 \nu^*_2}{w^*_2 \nu_2} - 1. \]
Using this result, one can verify that given market clearing, this result is consistent with the multipliers that we solved for above. Summarizing the second period policy (which is exactly the same as CDVW, augmented for our technology process):

\[ \tau_2 = \frac{w_2 \nu^*_2}{w_2 \nu_2} - 1 \]

\[ t^H_{2,x} = A^{-1} \left( \frac{\nu_2}{\nu^*_2} (1 + x) \right) \]

\[ t^H_2 = A^{-1} \left( \frac{\nu_2}{\nu^*_2} \right). \]

This gives us the problem of the planner for period 2 given the optimal policy in period 1. But how to determine the optimal policy in period 1? The Lagrangian is not differentiable in \( m_{i1} \), so relaxing the problem so that \( m_{i1} \in [0, 1] \) does not work. This is because \( m_{i1} \) changes the technology in the next period. Instead, we directly compare the choice of \( m_{i1} \) good by good, using the period 2 problem. The linearity of \( L \) in \( i \) greatly facilitates analysis. In particular, we can optimize good-by-good to solve this problem, taking account of the optimal choice of \( c_{i1}, m_{i2}, \) and \( m_{i2} \) conditional on \( m_{i1} \).

**Step 1: Solving for Cutoff Rules**

Let \( L_i^{m_{i1}, m_{i2}, m^*_{i2}} \) the contribution to the Lagrangian of good \( i \) given choices \( m_{i1}, m_{i2}, \) and \( m^*_{i2} \). One can show that given the optimal consumption decision,

\[
L_i^{m_{i1}, m_{i2}, m^*_{i2}} = \kappa_i - \beta_{i1} \log(a_{i1} \nu_1 (1 - m_{i1}) + a^*_{i1} \nu^*_1 m_{i1}) - \delta \beta_{i2} \log(a_{i2} \nu_2 (1 - m_{i2}) + a^*_{i2} \nu^*_1 m_{i2}) \\
- \delta \beta_{i2} [\nu^*_2 (1 - m^*_{i2}) + \nu_2 w^*_2 / w_2 m^*_{i2}] \tag{18}
\]

where \( \kappa_i \) is a constant independent of any choices. This formulation of the problem in the first period naturally lends itself to a solution strategy: every good \( i \) will be in some interval of the space partitioned by all of the cutoffs, \( \tau_i^H, \tau_2^F, \tau_2^H, \tau^H_{2,x}, \tau^F_{2,x}, \tau^H_2 \). These are formed by partitioning the unit interval according to the inverse image of \( A_i \) and \( A_i/(1 + x) \) at \( \nu_1/\nu^*_1, \omega_2, \) and \( \nu_2/\nu^*_2 \). Depending on which region good \( i \) is in, the planner will be making a binary choice between producing (by using a prohibitive tariff) or importing (by setting the uniform tariff), that depends on the outcomes in the second period. By way of example, consider the space carved out between \( \tau_2^F, x \) and \( \tau_2^H, x \). Here, if the planner uses a prohibitive tariff to produce in the first period, then in the second period, \( i \) will be to the left of both relevant cutoffs, \( \tau_2^F \) and \( \tau_2^H \), so Home will again produce and Foreign will import—leading to the ordered triple \((0, 0, 1)\). On the other hand, if the planner imports, in the next period it will still be optimal to produce since \( i \) is to the left of \( \tau_2^H, x \), even though the planner will operate the inferior technology; however, Foreign will not import since \( i \) would be to the right of \( \tau_2^F, x \)—hence the ordered triplet, \((1, 0, 0)\). We will assume that the cutoffs are ranked in the order above. However, by shifting \( \nu_1/\nu^*_1 \) down until \( \tau_i^H \) falls to the right of \( \tau_i^F, x \), it is clear that this need not be the case. However, this is the maximal number of partitions and for any smaller partition, the cutoffs solved
here would still work (but would occasionally be trivial—we will demonstrate an example below). With the partition of the space above, one can solve for the optimal first period strategy using this setup.

\[0, \iota_H^1\]

There is no internal disagreement with the planner in either period, nor will Foreign’s actions change, and so in this region the planner will always produce. Formally, the planner will use the same cutoff rule as in the next subsection, but it will always hold trivially for goods where \(A_i > \nu_1/\nu_i^*\).

\[\iota_H^1, \iota_F^1, \iota^2_F, \iota^2_H, x^2\]

In this region, the comparative advantage of Home is so high that, in the second period, even with the adjustment cost technology, Foreign will choose to import and Home will choose to produce. However, in the first period, this is suboptimal for Home (this is because the first period cutoff, ignoring the second period terms, would be at \(\iota_1^H\)). Comparing the planner’s options:

\[
L^{1,0.1} - L^{0,0.1} = \beta_{i1} \log \frac{a_i \nu_1}{a_i^1 \nu_i^1} + \delta \beta_{i2} \log \frac{a_i \nu_2}{a_i(1 + x) \nu_2}
\]

Simplifying the above expression yields the following cutoff:

\[
m_{i1} = 1 \iff A_i \leq \left(\frac{\nu_1}{\nu_i^1}\right) (1 + x)^\frac{a_i \beta_{i2}}{\mu_i^1}
\]

This says that the static cutoff is scaled by the second term. If \(x\) is larger, the cutoff shrinks, because this raises the cost of not producing in the second period. If \(\beta_{i1} \to 0\), then this goes to 0 (so you always produce, because very little labor is needed to cover production). If \(\delta \beta_{i2} \to 0\), you remain with the static cutoff.

\[\iota_F^1, \iota^2_F, \iota_F^2, x^2\]

In this region, Foreign will change their sourcing decision depending on Home’s actions. One can show by a similar calculation as before that,

\[
L^{1,0.0} - L^{0,0.1} = \beta_{i1} \log \frac{a_i \nu_1}{a_i^1 \nu_i^1} + \delta \beta_{i2} \log \frac{a_i \nu_2}{a_i(1 + x) \nu_2} + \delta \beta_{i2} L^* \left[\nu_2 w_2^* / w_2 - \nu_2^*\right].
\]

Rearranging, simplifying and using the definitions of the tariff yields the cutoff,

\[
m_{i1} = 1 \iff A_i \leq \left(\frac{\nu_1}{\nu_i^1}\right) (1 + x)^\frac{a_i \beta_{i2}}{\mu_i^1} \exp \left( - \frac{\delta \beta_{i2}}{\beta_{i1}} \times \frac{\tau_2}{1 + \tau_2} \times L^* \nu_i^2 \right)
\]

Notice that since period 2 actions depends on period 1 actions, and the first period cutoff depends on second period outcomes, the above cutoff is implicit and the solution to a fixed point.
In this region, both home and foreign will change their sourcing decision depending on what happens in the first period. In this interval, home will either produce the entirety of the good in period 2, or none of the good in period 2, depending on their action. Plugging in,

$$L^{1,1,0} - L^{0,0,1} = \beta_{i1} \log \frac{a_i \nu_1}{a_i^* \nu_1^*} + \delta \beta_{i2} \log \frac{a_i \nu_2}{a_i^* \nu_2^*} + \delta \beta_{i2} L^* \left[ \frac{\nu_2 w^*_2}{w_2} - \nu_2^* \right].$$

Rearranging and simplifying yields the cutoff,

$$m_i = 1 \iff A_i \leq \left( \frac{\nu_1}{\nu_1^*} \right)^{\frac{\delta \beta_{i2}}{\beta_{i1} + \delta \beta_{i2}}} \times \exp \left( -\frac{\delta \beta_{i2}}{1 + \tau_2} \times \frac{\tau_2}{1 + \tau_2} \times \frac{\nu_2}{\nu_2^*} \right).$$

This expression is similar to before, but now it does not depend on $x$ at all—this is because the inferior technology is never operated (but $x$ determines the size of this region). Instead, there is a weighted geometric mean between the two cutoffs. As before, we have a fixed point in the import share.

In this region, Foreign will always produce domestically, so there is no margin to induce Foreign to change their action. However, Home’s action in period 2 will depend on their action in period 1. Comparing the two situations,

$$L^{1,1,0} - L^{0,0,1} = \beta_{i1} \log \frac{a_i \nu_1}{a_i^* \nu_1^*} + \delta \beta_{i2} \log \frac{a_i \nu_2}{a_i^* \nu_2^*}.$$

This leads to the policy,

$$m_i = 1 \iff A_i \leq \left( \frac{\nu_1}{\nu_1^*} \right)^{\frac{\beta_{i1}}{\beta_{i1} + \delta \beta_{i2}}} \times \left( \frac{\nu_2}{\nu_2^*} \right)^{\frac{\delta \beta_{i2}}{\beta_{i1} + \delta \beta_{i2}}}.$$

This is a simple geometric weighted average of the two cutoff rules.

In this interval, the planner will always import regardless. Comparative advantage dominates all other concerns.

**Step 2: Solving for the Implicit Cutoffs**

Above we have described 6 regions and 6 cutoff rules. However, in 2 regions, these cutoffs are implicitly defined by the cutoff rules across all goods. That is to say, there is interdependence across the goods in this case. It is not obvious that a solution to this exists, such that the resulting multipliers (which are functions of the import shares) are consistent with the cutoff rules. Nevertheless, this is key to the solution, as it is necessary for us to be able to decentralize the problem. Here we offer a constructive proof that conditional
on \( w_t, w_t^\ast \), and \( \nu_t/\nu_t^\ast \), a solution exists and is unique. First, note that by plugging in the definition of \( \nu_2^\ast \), the relevant term can be written as \( \exp \left( a - b \frac{\Phi_H}{\Phi_F} \right) \), where \( \Phi_H \) is the home import share, \( \Phi_F \) is the foreign import share, and \( a \) and \( b \) are constants (that will depend on the multipliers and wages). Since \( m_{i2} \) is increasing in \( m_{i1} \) and \( m_{i2}^\ast \) is decreasing in \( m_{i1}^\ast \), \( \nu_2^\ast \) itself is monotonically increasing in the first period’s import share, dictated by \( m_{i1} \). Consider the operator \( T(\nu^\ast) \) which constructs the ratio of second period import shares using the cutoff rules with \( \nu^\ast \). This is monotonically decreasing in \( \nu^\ast \) by the argument above, since a higher \( \nu^\ast \) tightens the cutoffs for importing. Since \( T \) is monotonically decreasing, if it has a fixed point, will be a unique fixed point. It remains to prove this fixed point exists. To solve for this, notice that \( \nu^\ast \) is bounded above by \( 1/ \int_0^{\nu_2^\ast} \beta_{ij} \). That is to say, since wages are fixed, the least that Foreign will import is the import share if Home operated only the bad technology in period 1; similarly, the largest import share that Home can have is 1. Let \( \nu_0^\ast \) be this upper bound. Consider \( T(\nu_0^\ast) \). Since we have set the tightest possible bounds on importing activity, \( T(\nu_0^\ast) \) yields the smallest possible import share. Suppose this wasn’t the case and the optimal import share were smaller. Let \( i \) be a good that is imported in the case of \( T(\nu_0^\ast) \) but exported in the other case. Then \( A_i \) is above the tightest possible cutoff for importation, and so its importing could not possibly be optimal. By the same logic, \( T(T(\nu_0^\ast)) \) yields the largest possible import shares. However, since we have already constructed the largest possible import share, it must be that \( T(T(\nu_0^\ast)) \leq \nu_0^\ast \). One can continue this pattern iteratively: since success applications of \( T \) either tighten or loosen the constraints less than previous applications, \( T^2N(\nu_0^\ast) \) will be a decreasing sequence. This sequence is bounded below by 0 (autarky), and so must converge to a limit. Hence, \( \nu^\ast = \max \left( \frac{1}{\int_0^{\nu_2^\ast} \beta_{i2} \delta_i} \right) \) is the unique fixed point that solves this system.

Moreover, we can write the import shares in terms of the cutoff rules implicitly. As long as \( A \) is at least one differentiable everywhere and strictly monotone (so never \( A' = 0 \)), then by the implicit function theorem, the import shares will be differentiable and we can solve for their derivative. To give an example, suppose that in the \( x < \infty \) case, the cutoffs bind are interior in both the third and fourth regions. In this case,

A.2.1 Step 3: Sufficient Conditions for Existence

Given the definitions of the multipliers, it should be clear that a sufficient condition for a fixed point to the planner’s solution to exist would be if the optimal \( \nu_2/\nu_2^\ast \) were monotonically increasing in \( \nu_1/\nu_1^\ast \) and if import shares in the first period were monotonically increasing in \( \nu_1/\nu_1^\ast \). If this were the case, then the mapping \( T(\nu_1/\nu_1^\ast) \) onto itself that begins with a guess of the ratio of multipliers and returns back the multipliers as functions of the resulting import shares would be a monotonically increasing function. As it would be monotonically increasing, and it is clear that one could set \( \nu_1/\nu_1^\ast 1 = \omega_1 \) to achieve free trade and set \( \nu_1/\nu_1^\ast \) to achieve autarky, then there would be a fixed point between these two bounds. This is the same logic that works in DFS or CDVW, as can be seen in equation (8), specialized to our series of cutoffs.

Why is this not immediate in our case? In regions 3 and 4 of figure 4, the cutoffs contain an exponential term that is negative in the ratio of the import shares. Hence, while changing \( \nu_2/\nu_2^\ast \) may increase imports by
loosening the constraints, theoretically the exponential term could eventually dominate so much that imports begin to decline or remain flat. While we know the import shares exist as a function of $\nu_2/\nu_2^*$, we do not know necessarily if they increase or decrease. If the import shares are not monotonic, a fixed point may still exist—but guaranteeing it is harder, and a fixed point to $T$ may not exist. Instead of trying to find necessary conditions for existence of the fixed point, we will find sufficient conditions for the cutoffs to be strictly monotonically increasing in $\nu_2/\nu_2^*$. If this is the case, then there will be a fixed point in the second period multipliers conditional on $\nu_1/\nu_1^*$. Moreover, this fixed point will necessarily be (weakly) increasing in $\nu_1/\nu_1^*$, as the worse technology will more often be used. And then in turn we will have completed the proof as obviously the cutoffs are strictly increasing in $\nu_1/\nu_1^*$ otherwise.

Turning to the sufficient conditions, we will consider the “worst” case scenario—which is that the cutoffs in regions 3 and 4 are both turned on. If the multipliers are such that the constraint itself is nowhere binding in either region, then the proof is complete as these are the only cutoffs where problems can emerge.

In this case, home’s expenditure on imports (inclusive of tariffs), omitting time subscripts for clarity, is given by:

$$
\Phi_H = 1 - \lambda A^{-1}(C_3) + \lambda \left[ A^{-1} \left( \frac{\nu_2}{\nu_2^*} (1 + x) \right) - A^{-1}(C_3) \right] + \lambda \left[ A^{-1}(\omega_2(1 + x)) - A^{-1} \left( \frac{\nu_1}{\nu_1^*} (1 + x)^{-\delta \lambda} \right) \right], \quad (19)
$$

where $C_3$ and $C_4$ are the cutoffs in the third and fourth region, which contain the exponential term.

**Decentralizing the Problem**

We have established that there exists a unique solution for $m_{it}$, $c_{it}$, $\nu_i^*$, $\nu_i$, and $t_{it}$ that solves the relaxed problem. Moreover, we have shown that the solution fulfills all restrictions of the constrained problem. Hence, this must also be a solution of the constrained problem. One can also verify that at the solution, markets clear.

**Solving for the Optimal Tariff**

Given the results above, we can solve for $\tau_1$ and $\tau_2$ in a two step procedure. First, fix $\tau_1$ and $w_1$. Then, for a given $\tau_2$, solve for $w_2$ by successively iterating on the second period market clearing condition. This yields the equilibrium for a given value of $\tau_2$. Now one can optimize on this given $\tau_1$ and $w_1$. This yields $\tau_2$ as an implicit function of $\tau_1$, and one can then optimize over $\tau_1$. 

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